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Harvey Brooks

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Injection and Diffusion of Holes and Electrons in a Semiconductor[1]

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Abstract

The basic equations for the flow of electrons and holes in a semiconductor are presented and discussed. A small signal transient solution for an injected pulse of holes is obtained which is applicable to semiconductors which are intrinsic or nearly so, as well as to ordinary n and p types. Three conditions for the validity of the small signal solution are derived and discussed. In particular, a criterion for the validity of the assumption of space-charge neutrality is discussed in detail. A second approximation to take into account the beginning of nonlinear effects for large injected pulses is also derived and discussed. Finally, the theory of conductivity pulses due to injected carriers in a filament is discussed in the light of the preceding theory, and it is shown that the apparent quantum efficiency for photoconductivity in an intrinsic semiconductor can become very much larger than b + 1, where b is the mobility ratio.

I.

The Basic Equations

Using the mass-action law for recombination the diffusion of carriers in a homogeneous semiconductor may be described by the following system of equations:[2]

$$\frac{\partial n}{\partial t} = -r(np - n_i^2) + \frac{\partial}{\partial x}(\mu_n nE) + D_n \frac{\partial^2 n}{\partial x^2} \tag{1.1}$$

$$\frac{\partial p}{\partial t} = -r(np - n_i^2) - \frac{\partial}{\partial x}(\mu_p pE) + D_p \frac{\partial^2 p}{\partial x^2} \tag{1.2}$$

$$\frac{dE}{dx} = \frac{4\pi e}{K}(p - n + n_e) \tag{1.3}$$

where:

n = local density of electrons

p = local density of holes

μ_n, μ_p = mobility of electrons and holes, respectively

D_n, D_p = diffusion coefficient for electrons and holes, respectively

E = local electric field intensity

K = macroscopic dielectric constant

Equation (1.1) is the conservation equation for electrons, Eq. (1.2) that for holes, and Eq. (1.3) is Poisson's equation. In this report we consider only the one-dimensional problem, so that quantities n, p, and E, constituting the dependent variables, depend only on the single space coördinate x.

In this report we shall consider solutions of the Eqs. (1.1), (1.2), and (1.3) which are applicable to an infinite homogeneous medium, or to cases in which boundary conditions can be neglected. In particular, we shall be concerned with solutions corresponding to the injection of a pulse of minority carriers at the plane x = 0 and time t = 0 --the so-called indicial solutions. Since the system cannot be solved rigorously for arbitrary injected densities, we shall be concerned principally with small-signal solutions and the conditions of their validity.

II.

Simplification of the Basic Equations

Subtracting (1.2) from (1.1) we have the following relation:

$$\frac{\partial}{\partial t}(n - p) = \frac{\partial}{\partial x}\left[(\mu_n n + \mu_p p)E\right] + D_n \frac{\partial^2 n}{\partial x^2} - D_p \frac{\partial^2 p}{\partial x^2} \qquad (2.1)$$

Substituting for $\frac{\partial E}{\partial x}$ from (1.3) Eq. (2.1) can be put in the form:

$$\frac{\partial}{\partial t}(n - p) = E\frac{\partial}{\partial x}(\mu_n n + \mu_p p) + \frac{4\pi e(\mu_n n + \mu_p p)}{K}(p - n + n_e)$$
$$+ D_n \frac{\partial^2 n}{\partial x^2} - D_p \frac{\partial^2 p}{\partial x^2} \qquad (2.2)$$

In Eq. (2.2) the coefficient of $(p - n + n_e)$ is the reciprocal of the natural relaxation time:

$$\frac{1}{\tau_p} = \frac{4\pi\sigma}{K} \qquad (2.3)$$

where σ is the conductivity. Except for very steep pulses, this quantity is such that the corresponding term in (2.2) is extremely large compared with the remaining terms unless:

$$p - n + n_e = 0 , \qquad (2.4)$$

i.e., electrical neutrality obtains throughout the region.

The significance of Eq. (2.4) may be seen in another rather simple manner. First integrate Eq. (1.3) from $-\infty$ to $+\infty$, obtaining:

$$E_+ - E_- = \frac{4\pi e}{K} \int_{-\infty}^{+\infty} (p - n + n_e)\, dx \,, \qquad (2.5)$$

where E_+ and E_- are the values of the electric field at $+\infty$ and $-\infty$, and the right side of (2.5) now depends only on time. Next integrate (2.1) from $-\infty$ to $+\infty$ taking into account (2.5) and the fact that the semiconductor assumes its equilibrium properties at sufficiently large distances from the injection point at $x = 0$. Then we obtain:

$$\frac{\partial G}{\partial t} = \frac{4\pi\sigma_o}{K} G \quad ; \quad G = \int_{-\infty}^{+\infty} (p - n + n_e)\, dx \qquad (2.6)$$

where $\sigma_o = e(\mu_n n_o + \mu_p p_o)$

n_o, p_o = electron and hole densities at $\pm\infty$

Equation (2.6) has the solution:

$$G = G_o\, e^{-\frac{t}{\tau_\rho}} \qquad (2.7)$$

Thus an initial deviation from electrical neutrality disappears rapidly with the time constant given by Eq. (2.3). A more precise criterion will be derived below.

If we now assume the condition of neutrality, Eq. (2.4), Eq. (2.1) reduces to:

$$\frac{\partial}{\partial x} \left\{ (\mu_n n + \mu_p p)E + (D_n - D_p) \frac{\partial p}{\partial x} \right\} = 0 \,, \qquad (2.8)$$

which may be integrated immediately, giving:

$$(\mu_n n + \mu_p p)E + (D_n - D_p) \frac{\partial p}{\partial x} = I/e \,, \qquad (2.9)$$

where I, an integration constant which may depend on time, has the physical interpretation of the total current density.

Next, we divide Eq. (1.1) by $\mu_n n$, Eq. (1.2) by $\mu_p p$ and add, assuming space-charge neutrality.

$$\left(\frac{1}{\mu_n n} + \frac{1}{\mu_p p}\right) \left[\frac{\partial p}{\partial t} + r(np - n_i^2)\right] = E\left(\frac{1}{n} - \frac{1}{p}\right)\frac{\partial p}{\partial x} + \frac{kT}{e}\left(\frac{1}{n} + \frac{1}{p}\right)\frac{\partial^2 p}{\partial x^2} \quad (2.10)$$

where we have made use of the Einstein relation to obtain the last term. Solving (2.9) for E and substituting into Eq. (2.10) we can obtain an equation which involves only p as the dependent variable. This is:

$$\frac{\partial p}{\partial t} + r(np - n_i^2) = -\frac{I}{e}\frac{(n-p)\mu_n\mu_p}{(\mu_n n + \mu_p p)^2}\frac{\partial p}{\partial x} + \frac{(n-p)\mu_n\mu_p}{(\mu_n n + \mu_p p)^2}(D_n - D_p)\left(\frac{\partial p}{\partial x}\right)^2 + \frac{(n+p)\mu_n\mu_p}{(\mu_n n + \mu_p p)}\frac{kT}{e}\frac{\partial^2 p}{\partial x^2} \quad (2.11)$$

Equation (2.11) may be rewritten in somewhat simpler form by noting that:

$$\frac{\mu_n\mu_p}{\mu_n + \mu_p}\frac{\partial}{\partial x}\left\{\frac{I(n-p)}{e(\mu_n n + \mu_p p)} + \frac{(n-p)(D_p - D_n)}{(\mu_n n + \mu_p p)}\frac{\partial p}{\partial x}\right\}$$

$$= -\frac{I(n-p)\mu_n\mu_p}{e(\mu_n n + \mu_p p)^2}\frac{\partial p}{\partial x} + \frac{(n-p)(D_n - D_p)\mu_n\mu_p}{(\mu_n n + \mu_p p)^2}\left(\frac{\partial p}{\partial x}\right)^2 + \frac{(n-p)(\mu_p - \mu_n)}{(\mu_p + \mu_n)}\frac{\mu_n\mu_p}{(\mu_n n + \mu_p p)}\frac{kT}{e}\frac{\partial^2 p}{\partial x^2} \quad (2.12)$$

The first two terms on the right of Eq. (2.12) are the same as the first two terms on the right of (2.11). Thus, comparison of (2.11) and (2.12) leads to the following result:

$$\frac{\partial p}{\partial t} = -r\left[p(p+n_e) - n_i^2\right] + \frac{1}{b+1}\frac{\partial}{\partial x}\left[\frac{I/e - (b-1)D_p \frac{\partial p}{\partial x}}{1 + \frac{b+1}{b}\frac{p}{n_e}}\right]$$

$$+ \frac{2b}{b+1} D_p \frac{\partial^2 p}{\partial x^2} \tag{2.13}$$

where we have used the neutrality condition to express n in terms of p. In this equation:

$b = \mu_n/\mu_p$ = ratio of mobility of electrons to holes.

It is interesting to note that if the sample is intrinsic, corresponding to $n_e = 0$, the second term on the right of (2.13) vanishes, and the equation becomes linear except for the recombination term. In this case the effective diffusion coefficient is given by:

$$D' = \frac{2b}{b+1} D_p = \frac{2}{\frac{1}{D_p} + \frac{1}{D_n}} \tag{2.14}$$

If we set $I_p = 0$ in Eq. (2.13) and neglect recombination, the result is identical with that previously derived by Herring and quoted in Shockley's book (Eqs. 6 and 7, p.329).

It is evidently possible to include the last term in Eq. (2.13) in the brackets. When this is done, the equation takes the simple form:

$$\frac{\partial p}{\partial t} = -r\left[p(p+n_e) - n_i^2\right] + \frac{\partial}{\partial x}\left\{\frac{I/e(b+1) + (1 + \frac{2p}{n_e}) D_p \frac{\partial p}{\partial x}}{1 + \frac{b+1}{b}\frac{p}{n_e}}\right\} \tag{2.15}$$

Equation (2.15) is rigorous except for the assumption of the neutrality condition and the mass action recombination law. It is not possible to solve in this most general form because of its nonlinearity.

III.

Approximate Solutions of the Basic Equations

In order to solve (2.15), we assume

$$p = p_o + p_1 ,$$

where p_o is the equilibrium hole density, and expand the quantity in brackets of (2.15) as a Taylor Series in p_1. In this way we obtain:

$$\frac{\partial}{\partial x}\left\{ \frac{I/e(b+1)}{1+\frac{b+1}{b}\frac{p_o}{n_e}} - \frac{I/ebn_e}{\left[1+\frac{b+1}{b}\frac{p_o}{n_e}\right]^2} p_1 + \frac{1+\frac{2p_o}{n_e}}{1+\frac{b+1}{b}\frac{p_o}{n_e}} D_p \frac{\partial p_1}{\partial x} \right.$$

$$\left. + \frac{(I/eb)\frac{b+1}{b}}{\left[1+\frac{b+1}{b}\frac{p_o}{n_e}\right]^3}\left(\frac{p_1}{n_e}\right)^2 + \frac{\frac{b-1}{2b}}{\left[1+\frac{b+1}{b}\frac{p_o}{n_e}\right]^2}\frac{1}{n_e} D_p \frac{\partial p_1{}^2}{\partial x^2} \right\} \tag{3.1}$$

The last two terms in brackets are of second order compared with the first three. For example, the ratio of the fourth to the second term is:[3]

$$\frac{b+1}{b}\frac{p_1}{n_e+\frac{b+1}{b}p_o} = \frac{(\mu_n+\mu_p)p_1}{\mu_n n_o+\mu_p p_o} \ll 1 \tag{3.2}$$

This ratio is just the local percentage enhancement of conductivity due to the extra carriers. Provided the ratio is everywhere small, we are justified in dropping the fourth term. Similarly the ratio of the fifth to third term may be written:[3]

$$\frac{(\mu_n + \mu_p)p_1}{\mu_n n_o + \mu_p p_o} \cdot \frac{b-1}{b+1}\,\frac{n_o - p_o}{n_o + p_o} << 1 \qquad (3.3)$$

Equation (3.3) is always a weaker condition than (3.2) since the second factor is smaller than unity.

We can thus obtain an approximate solution of (2.15) by dropping the last two terms in the brackets of (3.1) The recombination term must also be linearized by neglecting terms of order p_1^2. Thus:

$$p(p + n_e) - n_i^2 = p_1^2 + p_1(n_o + p_o) \approx p_1(n_o + p_o), \qquad (3.4)$$

provided $$\frac{p_1}{n_o + p_o} << 1 .$$

The condition (3.4) is of about the same stringency as (3.2). Using the three approximations (3.2), (3.3), and (3.4) Eq. (2.15) may be written in the simple linear form:

$$\frac{\partial p_1}{\partial t} = -\frac{p_1}{\tau_p} - \mu' E_o \frac{\partial p_1}{\partial x} + D' \frac{\partial^2 p_1}{\partial x^2} \qquad (3.5)$$

where

$$\left.\begin{aligned}
\frac{1}{\tau_p} &= r(n_o + p_o) \\
E_o &= \frac{I}{e\mu_n n_e\left(1 + \frac{b+1}{b}\frac{p_o}{n_e}\right)} = \frac{I}{e(\mu_n n_o + \mu_p p_o)} \\
\mu' &= \frac{\mu_p}{\left[1 + \frac{b+1}{b}\frac{p_o}{n_e}\right]} = \frac{n_o - p_o}{\frac{n_o}{\mu_p} + \frac{p_o}{\mu_n}} \\
D' &= \frac{1 + \frac{2p_o}{n_e}}{\left[1 + \frac{b+1}{b}\frac{p_o}{n_e}\right]} D_p = \frac{n_o + p_o}{\frac{n_o}{D_p} + \frac{p_o}{D_n}}
\end{aligned}\right\} \qquad (3.6)$$

When $n_o >> p_o$, $\mu' \rightarrow \mu_p$ and $D' \rightarrow D_p$, while if $p_o >> n_o$, $\mu' \rightarrow -\mu_n$ and $D' \rightarrow D_n$, as would be expected from physical arguments.

It is interesting to observe that if we substitute the actual local densities n and p in the definitions of μ' and D', so that μ' and D' become spatially varying functions, then the rigorous equation (2.15) can be written in terms of μ' and D' as follows:

$$\frac{\partial p}{\partial t} = -r(np - n_i^2) + \frac{\partial}{\partial x}\left\{\frac{I}{e}\frac{\mu'}{\mu_n + \mu_p} + D'\frac{\partial p}{\partial x}\right\} \tag{3.7}$$

Returning to Eq. (3.5), we find for this the "indicial" solution:

$$p_1 = \frac{Q}{\sqrt{4\pi D' t}}\, e^{-\frac{t}{\tau_p}}\, e^{-\frac{(x - \mu' E_o t)^2}{4D't}} \tag{3.8}$$

where Q is the number of excess carriers introduced per unit cross section area at t = 0, x = 0. The solution (3.8) is formally identical with that usually given for the Haynes experiment[4], except that the mobility and diffusion coefficients are replaced by their effective values. The solution (3.8) always violates the conditions (3.2), (3.3), and (3.4) for sufficiently small t. However, in practice we can never inject an infinitely short pulse of carriers. If we inject a Gaussian pulse of the form:

$$I(x,t) = \delta(t)\,\frac{Q}{\sqrt{4\pi D' t_o}}\, e^{-\frac{x^2}{4D' t_o}} \tag{3.9}$$

so that the total number of injected carriers is still Q, the solution is precisely of the form (3.8), with

$$p_1 = \frac{Q}{\sqrt{4\pi D'(t+t_o)}} e^{-\frac{t}{\tau_p}} e^{-\frac{(x-\mu' E_o t)^2}{4D'(t+t_o)}} \qquad (3.10)$$

$$t \geqslant 0$$

Here t_o defines the spatial extension of the original injection. Under such conditions (3.10) is valid at all times provided

$$p_m = \frac{Q}{\sqrt{4\pi D' t_o}}$$

everywhere satisfies condition (3.2).

IV.

The Second Approximation

Returning to Eq. (3.1), it is possible to take into account the correction terms by substituting the solution (3.8) for the terms which are quadratic in p_1 and treating these terms as a known source function for the linear differential equation (3.5). In this treatment the integrals can only be evaluated as elementary functions if we neglect the recombination term in (3.5), which we shall accordingly do. The rather involved algebra is exhibited in Appendix A. We can express the final result in the following way. The complete solution for p is given by:

$$p = p_o + p_1 + p_2 \qquad (4.1)$$

where p_1 is the solution (3.8) and p_2 is given by:

$$\frac{p_2}{p_1} = \frac{Q}{\sqrt{4\pi D't}} \frac{1}{n_o + p_o} \frac{\mu'}{\mu_p} \left\{ -\frac{b+1}{b} \frac{eE_o\sqrt{4D't}}{kT} \frac{\sqrt{\pi}}{2} \mathrm{Erf}\left(\frac{S}{\sqrt{4D't}}\right) \right.$$

$$\left. + \frac{b-1}{b}\left[\frac{S}{\sqrt{4D't}} \frac{\sqrt{\pi}}{2} \mathrm{Erf}\left(\frac{S}{\sqrt{4D't}}\right) - \frac{1}{2} e^{-\frac{S^2}{4D't}} \right] \right\} \qquad (4.2)$$

$$S = x - \mu' E_o t$$

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

The first term in the brackets is odd in S and thus gives rise to a correction which makes the pulse unsymmetrical, the asymmetry being such as to shift the maximum to negative values of S. On the time axis, for a fixed x, (which is what is observed on an oscilloscope in the Haynes experiment), the maximum of the pulse is shifted towards longer times as the intensity of the pulse is increased. The second term in brackets is even in S and so leads to no shift in the maximum but only a symmetrical change of shape.

For small $(S/\sqrt{4D't})$, i.e., near the center of the pulse, the bracketed expression in (4.2) may be written:

$$\{\ \} = -\frac{b+1}{b}\frac{eE_oS}{kT} - \frac{b-1}{2b}\left(1 - \frac{3}{4}\frac{S^2}{4D't}\right) \qquad (4.3)$$

For large argument the corresponding expression is:

$$\{\ \} = \frac{\sqrt{\pi}}{2}\left\{ \mp \frac{b+1}{b}\frac{eE_o\sqrt{4D't}}{kT} \pm \frac{S}{\sqrt{4D't}} \right\} \qquad (4.4)$$

the upper and lower signs applying, respectively, for positive and negative S.

According to (4.3), we can calculate that the maximum of the distorted pulse occurs for

$$S = 2D't \frac{Q}{\sqrt{4\pi D't}} \frac{1}{n_o + p_o} \frac{\mu'}{\mu_p} \frac{b+1}{b} \frac{eE_o}{kT} \tag{4.5}$$

or for fixed x

$$t = \frac{x}{\mu' E_o} \left[1 + \frac{Q}{\sqrt{4\pi D't}} \frac{1}{n_o + p_o} \frac{D'}{D_p} \frac{b+1}{b} \right] \tag{4.6}$$

The quantity $\frac{Q}{\sqrt{4\pi D't}}$ is the maximum excess hole (or electron) density at the center of the pulse. Thus the criterion for a small correction term becomes:

$$\frac{p_m}{n_o + p_o} \frac{D'}{D_p} \cdot \frac{b+1}{b} = \frac{(\mu_n + \mu_p)p_m}{\mu_n n_o + \mu_p p_o} \ll 1 \tag{4.7}$$

This condition is identical with (3.2).

Equation (4.4) in conjunction with (4.2) shows that near the wings of the pulse, i.e., for $S/\sqrt{4D't} > 1$, the field-dependent term of (4.4) results in multiplying the amplitude of p_1 by a factor

$$1 + \frac{Q}{n_o + p_o} \frac{\mu'}{\mu_p} \frac{b+1}{b} \frac{eE_o\sqrt{\pi}}{2kT} \tag{4.8}$$

for negative S and dividing by the same constant factor for positive S, thus making the pulse steeper on its receding edge and less steep on its advancing edge. Herring[5] has given a simple physical interpretation of this effect in terms of the fact that the effective sweep field is lower in regions of higher carrier density.

The second term in brackets of (4.2) is even in S and in general leads to a blunting of the pulse. For small S the second term is negative while for large S it becomes positive. Physically this effect comes about because near the center of the pulse the semiconductor is more nearly intrinsic and therefore according to (3.6) the effective diffusion coefficient is larger near the center of the pulse than near the edges. Hence the pulse spreads somewhat more rapidly than would be indicated from the equilibrium diffusion constant. Alternatively, in a case where holes are the majority carrier, μ' changes sign and the pulse is sharper than it would normally be, because the extra holes required for space-charge neutralization inhibit diffusion. Reference to Eq. (4.4) would suggest that effects due to diffusion dominate the mobility effect if S is sufficiently large. The condition for this, however, is that:

$$S > 4\,\frac{b+1}{b}\,\frac{n_o + p_o}{n_o - p_o}\,x\;; \tag{4.9}$$

i.e., at distances beyond the distance through which the pulse has been swept by the field. Thus, in fact, the effect represented by (4.8) dominates the situation under most conditions.

The factor (4.8) can be written approximately in the form:

$$1 + \frac{p_m(\mu_n + \mu_p)}{\mu_n n_o + \mu_p p_o}\;\cdot\;\left[\frac{eE_o x}{kT}\,\frac{n_o - p_o}{n_o + p_o}\right]^{\frac{1}{2}} \tag{4.10}$$

The correction term is to be compared with (4.7), and is seen to provide a somewhat more stringent condition, since the factor in brackets is usually appreciably greater than unity. In fact, the validity of the customary interpretation of the

Haynes experiment depends upon the condition that the factor in square brackets is much greater than 1, as is demonstrated in Appendix B. In a nonintrinsic experiment this factor is just the ratio of the voltage difference between emitter and collector point in the sweeping field to kT.

V.

The Neutrality Approximation

In Eq. (2.2) we saw that the term having the coefficient $\frac{4\pi\sigma}{K}$ was dominant. We may obtain a measure of the degree of violation of the neutrality condition by assuming $p - n + n_e = 0$ in the other terms of (2.2), which have much smaller coefficients. We thus obtain:

$$(p-n+n_e) = -\frac{K}{4\pi\sigma}\left\{(\mu_n+\mu_p)E\frac{\partial p}{\partial x} + (D_n - D_p)\frac{\partial^2 p}{\partial x^2}\right\} \tag{5.1}$$

Substituting for E in (5.1) from (2.9) we have:

$$n-p-n_e = \frac{K}{4\pi\sigma}\left\{\frac{(\mu_n+\mu_p)\,I}{e(\mu_n n+\mu_p p)}\left(\frac{\partial p}{\partial x}\right) - \frac{(D_n - D_p)(\mu_n+\mu_p)}{\mu_n n+\mu_p p}\left(\frac{\partial p}{\partial x}\right)^2 + (D_n - D_p)\frac{\partial^2 p}{\partial x^2}\right\}$$

$$= \frac{K}{4\pi e}\frac{\partial}{\partial x}\left\{-\frac{I}{e(\mu_n n+\mu_p p)} + \frac{D_n - D_p}{\mu_n n+\mu_p p}\frac{\partial p}{\partial x}\right\} \tag{5.2}$$

Comparing (5.2) with (2.13) we obtain, neglecting recombination:

$$n-p-n_e = \frac{K}{4\pi e \mu_n n_e} \left\{ 2D_n \frac{\partial^2 p}{\partial x^2} - (b+1) \frac{\partial p}{\partial t} \right\}$$

$$= \frac{K}{4\pi e n_e} \left(\frac{1}{\mu_n} + \frac{1}{\mu_p} \right) \left\{ \frac{2D_p D_n}{D_p + D_n} \frac{\partial^2 p}{\partial x^2} - \frac{\partial p}{\partial t} \right\} \quad . \qquad (5.3)$$

In terms of the basic solution (3.8) expressed in terms of S, Eq. (5.3) may be written:

$$\frac{n-p-n_e}{p_1} = \frac{K}{4\pi e n_e} \left(\frac{1}{\mu_n} + \frac{1}{\mu_p} \right) \left\{ \frac{2D_n D_p}{D_p + D_n} \frac{1}{p_1} \frac{\partial^2 p_1}{\partial S^2} + \mu' E_o \frac{\partial \ln p_1}{\partial S} - \frac{\partial \ln p_1}{\partial t} \right.$$

$$= - \frac{K}{4\pi e} \cdot \frac{1}{n_o + p_o} \cdot \frac{\mu_n + \mu_p}{\mu_n \mu_p} \cdot \frac{1}{2t} \left\{ \frac{eE_o S}{kT} + \frac{b-1}{b+1} \left[1 - \frac{S^2}{2D' t} \right] \right\}$$

$$= - \frac{bn_o + p_o}{n_o + p_o} \frac{K}{4\pi \sigma t} \left\{ \frac{b+1}{2b} \frac{eE_o S}{kT} + \frac{b-1}{2b} \left(1 - \frac{S^2}{2D' t} \right) \right\} \qquad (5.4)$$

We see from Eq. (5.4) that the principal criterion of neutrality is simply that

$$t >> \tau_\rho = \frac{K}{4\pi\sigma} = \frac{K\rho}{4\pi} \qquad (5.5)$$

or in practical units

$$t >> 0.885\ K\rho \times 10^{-14} \text{ sec.}$$

Since we are dealing with pulses whose length is measured in microseconds, we do not have to worry about neutrality until we have resistivities of the order of 10^8 ohm cm, of no practical importance in semiconductors. If one type of carrier, eg., holes, can be trapped, its effective mobility can become extremely low, effectively increasing b in Eq. (5.4). Then b is replaced by b/f, where f is the fraction of time a hole spends trapped, and the condition (5.5) becomes weakened by a factor $1/f$ on the right. Thus for $f = 10^{-8}$, space-charge questions again become important, and it actually follows from (5.4) that the neutrality condition is violated if either carrier has its mobility sufficiently reduced by trapping. For Ge and Si at room temperature (5.5) is satisfied by a tremendous margin.

VI.

Photoconductivity Pulses (6)

Consider a filament with constant voltage applied along it. The voltage drop may be obtained from Eq. (2.9), as:

$$V_o = \int E dx = \int \frac{dx}{\sigma} - e(D_n - D_p) \int \frac{\partial p/\partial x}{\sigma} dx$$

$$= I \int \frac{dx}{\sigma} - e \frac{D_n - D_p}{\mu_n + \mu_p} \int \frac{\partial}{\partial x} \ln \sigma \, dx \qquad (6.1)$$

The integration over x is taken between electrodes. If we assume that electrodes maintain equilibrium charge densities in their vicinity, i.e., are ohmic in character, then it follows that the second term on the right of (6.1) vanishes. For small disturbances we have:

$$\frac{1}{\sigma} = \frac{1}{\sigma_o} - \frac{\Delta\sigma}{\sigma_o^2} = \frac{1}{\sigma_o} - \frac{\mu_n + \mu_p}{\sigma_o{}^2} p_1 , \qquad (6.2)$$

where σ_o = equilibrium conductivity and p_1 is the solution (3.8) of the generalized diffusion equation (3.5). Using (6.2) in (6.1) and transposing, we obtain:

$$I(t) = \frac{\sigma_o}{L} V_o + \frac{V_o}{L} (\mu_n + \mu_p) \frac{1}{L} \int p_1 dx \, , \tag{6.3}$$

where L = length of the filament. The first term is the equilibrium conduction current while the second term is increment of current arising from the injected carriers. More rigorously, p_1 should be not Eq. (3.8) but rather a modified solution which satisfies the boundary conditions $p_1 = 0$ at the ends of the filament. However, we may study the behavior qualitatively in terms of (3.8) The second term in (6.3) may be approximated by:

$$\begin{aligned} \int p_1 dx &= Q \gtrsim \quad 0 \lesssim t \lesssim \frac{L_1}{\mu' E_o} \\ &= 0 \qquad\quad t > \frac{L_1}{\mu' E_o} \; . \end{aligned} \tag{6.4}$$

Thus we have a current pulse which lasts until the excess carriers are swept to the electrode, a well-known result which applies if the lifetime τ_p is sufficiently long. The total charge pulse is thus of order:

$$\begin{aligned} \int_0^\infty \Delta I(t)\, dt &\simeq \frac{V_o}{L} (\mu_n + \mu_p) \cdot \frac{1}{L} \cdot Q \cdot \frac{L_1}{\mu' E_o} \\ &= \frac{\mu_n + \mu_p}{\mu'} \cdot \frac{L_1}{L} \cdot Q \, , \end{aligned} \tag{6.5}$$

where Q is the originally injected hole charge and L_1 is the distance between the injection point and the collecting electrode. According to (6.5) there is a charge amplification given by:

$$a = \frac{b+1}{b} \cdot \frac{bn_o + p_o}{n_o - p_o} \cdot \frac{L_1}{L} \; . \tag{6.6}$$

In the nearly intrinsic case, with μ' very small and the diffusion length long compared with the dimensions of the filament, we obtain:

$$\alpha = \frac{eV_o}{kT} \cdot \frac{L_1}{L} \frac{L_2}{L} \frac{b+1}{b} \frac{bn_o + p_o}{n_o + p_o} \quad . \tag{6.7}$$

For uniform illumination of the filament this becomes:

$$\alpha = \frac{eV_o}{6kT} \frac{b+1}{b} \frac{bn_o + p_o}{n_o + p_o} \tag{6.8}$$

The condition for the validity of this expression is that:

$$\frac{eV_o}{kT} \frac{L_2}{L} << \frac{n_o + p_o}{n_o - p_o} \quad , \tag{6.9}$$

whence an upper limit to α is:

$$\alpha << \frac{L_1}{L} \frac{bn_o + p_o}{n_o - p_o} \frac{b+1}{b} \quad , \tag{6.10}$$

which shows that (6.7) makes a smooth transition into (6.6) as the sweeping voltage is increased.

Another case of interest is that in which the diffusion length is less than the distance to electrodes. Then Eq. (C.7) of the appendix leads to:

$$\alpha = \frac{V_o}{L} \frac{(\mu_n + \mu_p)\tau}{L} \quad . \tag{6.11}$$

Provided recombination and trapping can be neglected, Eq. (6.8) is applicable even when the filament is nearly an insulator.

It then becomes:

$$\alpha = \frac{eV_o}{6kT} \frac{(b+1)^2}{b} , \qquad (6.12)$$

since the insulator becomes in effect an intrinsic semiconductor in the illuminated region. Equation (6.12) shows that it is possible to have apparent quantum efficiencies substantially greater than 1 in an intrinsic semiconductor or in a semiconductor which is made locally intrinsic by high radiation intensity. Eq. (6.12) is only valid for an insulator, however, if electrons are free to flow into the conduction band of the insulator from the electrodes.

Appendix A

From Eq. (3.5) with $\tau_p = \infty$ and Eq. (3.1), the inhomogeneous linear differential equation to be solved may be written:

$$\frac{\partial p_2}{\partial t} = - \mu' E_o \frac{\partial p_2}{\partial x} + D' \frac{\partial^2 p_2}{\partial x^2} \tag{A.1}$$

$$+ \frac{\partial}{\partial x} \left[\frac{(I/eb)\,\frac{b+1}{b}}{\left[1 + \frac{b+1}{b}\frac{p_o}{n_e}\right]^3} \left(\frac{p_1}{n_e}\right)^2 + \frac{\frac{b-1}{2b}}{\left[1 + \frac{b+1}{b}\frac{p_o}{n_e}\right]^2} \frac{1}{n_e} D_p \frac{\partial p_1^2}{\partial x} \right],$$

where p_1 is given by (3.8). Changing variables to:

$$S = x - \mu' E_o t$$

$$t' = D' t$$

and making use of the definitions of μ' and D' in (3.5), Eq. (A.1) reduces to:

$$\frac{\partial p_2}{\partial t'} = \frac{\partial^2 p_2}{\partial S^2} + \frac{1}{D'} \frac{\partial}{\partial S} \left[E_o \frac{\mu'^2}{\mu_p} \frac{p_1^2}{n_e} \frac{b+1}{b} + D' \frac{\mu'}{\mu_p} \frac{1}{n_o + p_o} \frac{\partial p_1^2}{\partial S} \frac{b-1}{2b} \right] \tag{A.2}$$

Treating the last term as a source, this equation has the solution:

$$p_2 = \int_{-\infty}^{+\infty} d\sigma \int_{-\infty}^{t'} d\tau \; \sum(\sigma,\tau) \frac{1}{\sqrt{4\pi(t'-\tau)}} \, e^{-\frac{(S-\sigma)^2}{4(t'-\tau)}}$$

(A.3)

$$\sum(\sigma,\tau) = \frac{\mu'}{\mu_p}\frac{\partial}{\partial\sigma}\left[E_o \frac{\mu'}{D'}\frac{p_1^2}{n_e}\frac{b+1}{b} + \frac{1}{n_o+p_o}\frac{\partial p_1^2}{\partial\sigma}\frac{b-1}{2b}\right]$$

By a simple change of variable and one integration by parts, Eq. (A.3) can be put in the form:

$$p_2 = -\int_{-\infty}^{+\infty} d\sigma \int_{0}^{t'} d\tau \; f(S-\sigma,t'-\tau)\frac{1}{\sqrt{4\pi\tau}}\frac{\sigma}{2\tau}\, e^{-\frac{\sigma^2}{4\tau}} \qquad \text{(A.4)}$$

$$\text{with} \quad f(S-\sigma,t'-\tau) = \frac{\mu'}{\mu_p}\left[\frac{eE_o}{kT}\frac{b+1}{b}p_1^2 - \frac{b-1}{2b}\frac{\partial p_1^2}{\partial\sigma}\right]\frac{1}{n_o+p_o},$$

where we have used the relation:

$$\frac{\mu'}{D'} = \frac{e}{kT}\frac{n_o-p_o}{n_o+p_o} \qquad \text{(A.5)}$$

In Eq. (A.4) p_1^2 is understood to be expressed as a function of the variable $S-\sigma$ and $t'-\tau$ as indicated--hence the change in sign of the derivative term.

To simplify (A.4) still further we integrate the second term by parts with respect to σ. The result is:

$$p_2 = -\frac{\mu'}{\mu_p}\frac{1}{n_o+p_o}\int_{-\infty}^{+\infty} d\sigma \int_{0}^{t'} d\tau \, \frac{p_1^2}{\sqrt{4\pi\tau}}\left[\frac{eE_o}{kT}\frac{b+1}{b}\frac{\sigma}{2\tau} + \frac{b-1}{2b}\left(\frac{1}{2\tau} - \frac{\sigma^2}{4\tau^2}\right)\right] e^{-\frac{\sigma^2}{4\tau}}$$

(A.6)

Written out explicitly this integral becomes:

$$p_2 = - \frac{\mu'}{\mu_p} \cdot \frac{1}{n_o + p_o}$$

$$\cdot\; Q^2 \int_{-\infty}^{+\infty} d\sigma \int_0^{t'} d\tau \; \frac{1}{\sqrt{4\pi\tau}} \; \frac{1}{4\pi(t'-\tau)} \left[\frac{eE_o}{kT}\,\frac{b+1}{b}\,\sigma + \frac{b-1}{2b}\left(1-\frac{\sigma^2}{2\tau}\right)\right] \frac{1}{2}$$

$$\cdot\; e^{-\frac{\sigma^2}{4\tau}}\, e^{-\frac{(S-\sigma)^2}{2(t'-\tau)}} \,. \tag{A.7}$$

We carry out the integration over σ first, noting that:

$$\frac{\sigma^2}{4\tau} + \frac{(S-\sigma)^2}{2(t'-\tau)} = \frac{1}{4\tau}\,\frac{t'+\tau}{t'-\tau}\left(\sigma - \frac{2\tau}{t'+\tau}\,S\right)^2 + \frac{S^2}{2(t'+\tau)} \tag{A.8}$$

Equation (A.8) suggests changing the integration variable to:

$$\xi = \frac{1}{2\sqrt{\tau}}\sqrt{\frac{t'+\tau}{t'-\tau}}\left(\sigma - \frac{2\tau}{t'+\tau}\,S\right)$$

$$\frac{\sigma}{2\tau} = \frac{S}{t'+\tau} + \frac{\xi}{\sqrt{\tau}}\sqrt{\frac{t'-\tau}{t'+\tau}} \tag{A.9}$$

Substituting (A.9) in (A.7) we may drop immediately the terms which are odd in ξ, and we are left with:

$$p_2 = - \frac{\mu'}{\mu_p}\,\frac{Q^2}{n_o + p_o} \int_{-\infty}^{+\infty} d\xi \int_0^{t'} d\tau \; e^{-\frac{S^2}{2(t'+\tau)}} \tag{A.10}$$

$$\left[\frac{eE_o}{kT}\,\frac{b+1}{b}\,\frac{S}{t'+\tau} + \frac{b-1}{2b}\left(\frac{1}{2\tau} - \frac{S^2}{(t'+\tau)^2} - \frac{\xi^2}{\tau}\,\frac{t'-\tau}{t'+\tau}\right)\right] \cdot e^{-\xi^2}\, \frac{1}{4\pi}\,\frac{1}{\sqrt{t'^2-\tau^2}}\,\frac{1}{\sqrt{\pi}}$$

The integration over ξ gives immediately:

$$p_2 = - \frac{\mu'}{\mu_p} \frac{Q^2}{n_o + p_o} \int_0^{t'} \frac{d\tau}{4\pi} e^{-\frac{S^2}{2(t'+\tau)}}$$

$$\cdot \left[\frac{eE_oS}{kT} \frac{b+1}{b} + \frac{b-1}{2b} \left(1 - \frac{S^2}{t'+\tau}\right) \right] \sqrt{\frac{t'+\tau}{t' \cdot \tau}} \frac{d\tau}{(t'+\tau)^2} \qquad \text{(A.11)}$$

To integrate (A.11) we next change the variable of integration to

$$u = \frac{1}{2(t'+\tau)} ,$$

obtaining:

$$p_2 = - \frac{\mu'}{\mu_p} \frac{Q^2}{n_o + p_o} \cdot \frac{1}{2\pi} \int_{\frac{1}{4t'}}^{\frac{1}{2t'}} du \; e^{-uS^2}$$

$$\cdot \left[\frac{eE_oS}{kT} \cdot \frac{b+1}{b} + \frac{b-1}{2b} (1 - 2uS^2) \right] \frac{1}{\sqrt{4ut-1}} \qquad \text{(A.12)}$$

Still another change of variable is necessary to

$$v = 4ut - 1$$

with this substitution (A.12) transforms once again to:

The integration over ξ gives immediately:

$$p_2 = -\frac{\mu'}{\mu_p}\frac{Q^2}{n_o + p_o}\int_0^{t'} \frac{d\tau}{4\pi}\, e^{-\frac{S^2}{2(t'+\tau)}}$$

$$\cdot \left[\frac{eE_oS}{kT}\frac{b+1}{b} + \frac{b-1}{2b}\left(1 - \frac{S^2}{t'+\tau}\right)\right]\sqrt{\frac{t'+\tau}{t'-\tau}}\,\frac{d\tau}{(t'+\tau)^2} \qquad \text{(A.11)}$$

To integrate (A.11) we next change the variable of integration to

$$u = \frac{1}{2(t'+\tau)} ,$$

obtaining:

$$p_2 = -\frac{\mu'}{\mu_p}\frac{Q^2}{n_o + p_o}\cdot\frac{1}{2\pi}\int_{\frac{1}{4t'}}^{\frac{1}{2t'}} du\; e^{-uS^2}$$

$$\cdot \left[\frac{eE_oS}{kT}\cdot\frac{b+1}{b}+\frac{b-1}{2b}\left(1 - 2uS^2\right)\right]\frac{1}{\sqrt{4ut-1}} \qquad \text{(A.12)}$$

Still another change of variable is necessary to

$$v = 4ut - 1$$

with this substitution (A.12) transforms once again to:

$$p_2 = -\frac{\mu'}{\mu_p}\frac{Q^2}{n_o + p_o}\frac{e^{-\frac{S^2}{4t'}}}{4\pi t'}\int_0^1 e^{-\frac{S^2}{4t'}v}$$

$$\cdot\left[\frac{eE_oS}{kT}\frac{b+1}{b} + \frac{b-1}{2b}\left(1 - \frac{S^2}{2t'} - \frac{S^2}{2t'}v\right)\right]\frac{dv}{2v^{1/2}} \qquad (A.13)$$

Finally changing to $x^2 = \frac{S^2}{4t'}v$, we have:

$$p_2 = -\frac{\mu'}{\mu_p}\frac{Q^2}{n_o + p_o}\frac{e^{-\frac{S^2}{4t'}}}{\sqrt{4\pi t'}}\frac{1}{\sqrt{\pi}}\int_0^{\frac{S}{2\sqrt{t'}}} dx\, e^{-x^2}$$

$$\cdot\left[\frac{eE_o}{kT}\frac{b+1}{b} - \frac{S}{4t'}\frac{b-1}{b} + \frac{b-1}{2b}\frac{1}{S}(1 - 2x^2)\right] \qquad (A.14)$$

The result (4.2) follows immediately by integration of (A.14).

Appendix B

Consider the solution (3.8). In normal operation of the Haynes experiment the maximum of the pulse is determined from the condition:

$$\frac{\partial p_1}{\partial t} = 0 \tag{B.1}$$

This leads to:

$$-\frac{1}{2t} - \frac{1}{\tau_p} + \frac{(x-\mu' E_o t)^2}{4D' t^2} + \frac{\mu' E_o (x-\mu' E_o t)}{4D' t} = 0 \tag{B.2}$$

Let this be solved by

$$t = \frac{x}{\mu' E_o} + t_1 \tag{B.3}$$

where t_1 is treated as first-order small. Then we have:

$$-\frac{\mu' E_o}{2x} + \frac{1}{2}\left(\frac{\mu' E_o}{x}\right)^2 t_1 - \frac{1}{\tau_p} + \frac{(\mu' E_o)^2}{4D' x}(-\mu' E_o) t_1 = 0$$

or

$$t = \frac{x}{\mu' E_o}\left\{ 1 - \frac{1 + \frac{2x}{\mu' E_o}\frac{1}{\tau_p}}{\frac{\mu' E_o x}{2D'} - 1} \right\} \tag{B.4}$$

The mobility is usually estimated from (B.3) with $t_1 = 0$. The correction term in (B.4) is negligible only if

$$\frac{2x}{\mu' E_o} \text{ is not} \gg \tau_p$$

and
$$\frac{\mu' E_o x}{2D'} = \frac{eE_o x}{kT}\,\frac{n_o - p_o}{n_o + p_o} \gg 1 \quad . \tag{B.5}$$

The left-hand side of the inequality is the same as the multiplying factor in (4.10). It is interesting to observe that if we combine the (4.6) and (B.5) corrections we obtain:

$$t = \frac{x}{\mu' E_o}\left[1 + \frac{(\mu_n + \mu_p)p_m}{\mu_n n_o + \mu_p p_o} - \frac{kT}{eE_o x}\,\frac{n_o + p_o}{n_o - p_o}\right] \tag{B.6}$$

Appendix C

We assume that injection occurs at $x = 0$ and label the region $x > 0$ with subscript 1 and to the left with subscript 2. The quantity $\int_0^\infty p_1 dt$ satisfies the steady-state diffusion equation and so has the general solution:

$$x > 0 \qquad A_1 e^{-p_1 x} + B_1 e^{p_2 x}$$

$$x < 0 \qquad B_2 e^{-p_1 x} + A_2 e^{p_2 x}$$

$$p_1 = \sqrt{\left(\frac{\mu' E_o}{2D'}\right)^2 + \frac{1}{D'\tau}} - \frac{\mu' E_o}{2D'} \tag{C.1}$$

$$p_2 = \sqrt{\left(\frac{\mu' E_o}{2D'}\right)^2 + \frac{1}{D'\tau}} + \frac{\mu' E_o}{2D'}$$

Boundary conditions at the electrodes and at $x = 0$ give the following relations:

$$A_1 + B_1 = A_2 + B_2 \tag{C.2}$$

$$A_1 e^{-p_1 L_1} + B_1 e^{p_2 L_1} = 0 \tag{C.3}$$

$$B_2 e^{p_1 L_2} + A_2 e^{-p_2 L_2} = 0 \tag{C.4}$$

$$D'A_1 p_1 - D'B_1 p_2 - D'B_2 p_1 + D'A_2 p_2 = Q \tag{C.5}$$

From these relations we obtain:

$$B_1 = -A_1 e^{-(p_1+p_2)L_1}$$

$$B_2 = -A_2 e^{-(p_1+p_2)L_2}$$

$$A_2 = \frac{1-e^{-(p_1+p_2)L_1}}{1-e^{-(p_1+p_2)L_2}} A_1 \tag{C.6}$$

$$A_1 = \frac{Q}{D'} \cdot \frac{1}{p_1+p_2} \cdot \frac{1-e^{-(p_1+p_2)L_2}}{1-e^{-(p_1+p_2)(L_1+L_2)}}$$

$$\int_{-L_2}^{+L_1} Ap\,dx = \frac{Q}{D'} \cdot \frac{1}{p_1 p_2}$$

$$\cdot \left\{ \frac{\left(1-e^{-p_1L_1}\right)\left(1-e^{-p_2L_2}\right) - e^{-p_1L_1} e^{-p_2L_2}\left(1-e^{-p_2L_1}\right)\left(1-e^{-p_1L_2}\right)}{1-e^{-(p_1+p_2)(L_1+L_2)}} \right\}$$

$$= Q\tau \left\{ \frac{\left(e^{p_1L_1}-1\right)\left(e^{p_2L_2}-1\right) - \left(1-e^{-p_2L_1}\right)\left(1-e^{-p_1L_2}\right)}{e^{p_1L_1}e^{p_2L_2} - e^{-p_1L_1}e^{-p_2L_2}} \right\} \tag{C.7}$$

If $L_2 \gg L_1$ or $p_2L_2 \gg 1$ Eq. (C.7) reduces to:

$$Q\tau\left(1-e^{-p_1L_1}\right) \tag{C.8}$$

If $\mu' E_o \gg \left(\frac{2D'}{\tau}\right)^{\frac{1}{2}}$, we have:

$$p_1 = \frac{1}{\mu' E_o \tau} \quad . \tag{C.9}$$

For $\frac{L_1}{\mu' E_o \tau} << 1$, we obtain the result (6.6) directly.

Another result of interest occurs when both $p_1 L_1$ and $p_2 L_2$ may be regarded as small. This occurs in the nearly intrinsic case. Then the bracket in (C.7) reduces to

$$\frac{1}{2} p_1 p_2 L_1 L_2 = \frac{L_1 L_2}{2D'\tau} \quad . \tag{C.10}$$

Substitution in (6.3) results in Eq. (6.7).

Footnotes

1. Results very similar to these have been derived independertly by W. Van Roosbroek, to be published in Bell Syst. Tech. J. I am indebted to Dr. Van Roosbroek for several valuable discussions and for a pre-publication copy of his manuscript. These results were reported by the present author at the Cambridge meeting of the American Physical Society, January, 1953.

2. Various forms of these equations are discussed in W. Van Roosbroek, Bell Syst. Tech. J. 29 (4), 560 (1950), also by: W. Shockley, Electrons and Holes in Semiconductors, Van Nostrand (1950), pp. 318-320.

3. This criterion is also given in reference (1).

4. J. R. Haynes and W. Shockley, Phys. Rev. 75, 691 (1949).

5. C. Herring, Bell Syst. Tech. J. 28, 401 (1949) or Monograph No. 1726, p. 67.

6. Cf. also W. Van Roosbroek, J. App. Phys. 23, 1411 (1952)